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ERROR BOUNDS AND ASYMPTOTIC PERFORMANCE UNDER MISMATCH OF MULTISENSOR DETECTION SYSTEMS

Syracuse University

D. Kazakos

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Rome Air Development Center
Air Force Systems Command
Griffiss Air Force Base, NY 13441-5700

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APPROVED:

VINCENT C. VANNICOLA
Project Engineer

APPROVED:

FRED J. DEMMA
Acting Director of Surveillance

FOR THE COMMANDER:

JOHN A. RITZ
Directorate of Plans & Programs

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D. Kazakos

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Syracuse University
Department of Electrical Engineering
Syracuse NY 13244-1240

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A binary detection problem, when the data are obtained from distant sensors, and modelled as stationary Gaussion processes, with different spectra is considered. It is also assumed that inaccurate versions of the true statistical models are utilized, and upper bounds to the probability of error, based on the Chernoff bounding approach are developed. Those bounds also converge to the asymptotic probability of error as the number of data points increased. Conditions for sustaining, in spite of mismatch, exponential convergence of the error probability to zero are also determined.

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ERROR BOUNDS AND ASYMPTOTIC PERFORMANCE UNDER MISMATCH OF MULTISENSOR DETECTION SYSTEMS

D. Kazakos
Electrical Engineering Department
University of Virginia
Charlottesville, VA 22901

ABSTRACT

We consider a binary detection problem, when the data are obtained from m distant sensors, and modelled as stationary Gaussion processes, with different spectra. We also assume that inaccurate versions of the true statistical models are utilized, and we develop upper bounds to the probability of error, based on the Chernoff bounding approach. Those bounds also converge to the asymptotic probability of error as the number n of data points increases to ∞. Conditions for sustaining, in spite of mismatch, exponential convergence of the error probability to zero with n are determined.

INTRODUCTION

There has been much interest recently in the signal detection problem for data available from multiple sensors [1] - [7]. In this paper we develop bounds to the probability of error for binary detection from multisensor data, when inaccurate versions of the actual statistics are incorporated into the decision rule. We specifically investigate the error probability for detection in Gaussian, stationary processes with inaccurately known spectra.

SUMMARY OF RESULTS

Suppose that m sensors are utilized for deciding between two hypotheses, H_1, H_0. Let $x_k^n = (x_{k1} \cdots x_{kn})$ be the data vector for the kth sensor, distributed according to the probability density function $f_{jk}(x_k^n)$ for j=0,1 under H_0, H_1, correspondingly. Suppose, also, that $g_{jk}(x_k^n)$ is the "inaccurate" version of $f_{jk}(x_k^n)$ that is used in the decision rule. We will assume throughout this paper that the data from different sensors are statistically independent. This assumption will be removed in a subsequent paper. Because of the independence between distinct sensors, the likelihood ratio test is:

$$\text{Decide } H_1 \text{ if } m^{-1} \cdot \sum_{k=1}^{m} q_k(x_k^n) > T \tag{1}$$

where:

$$q_k(x_k^n) = n^{-1} \cdot \log \frac{g_{1k}(x_k^n)}{g_{0k}(x_k^n)} \tag{2}$$

is the "mismatched" log likelihood function of the kth sensor.

Let, for j=0,1:

$P_j(f,g,n) =$ Probability of erroneously deciding H_j using $\{g_{jk}\}$ and based on n measurements.

We will utilize the Chernoff bounding technique, and subsequently relate the bound to "large deviation theory," [8], [9], as $n \to \infty$.

The basic Chernoff bound is:

$$\begin{aligned} P_0(f,g,n) &= \Pr[m^{-1} \sum_{k=1}^{m} z_k > T \mid H_0] \\ &\le E_0 \exp[t(m^{-1} \sum_{k=1}^{m} z_k - T)] = \\ &= \prod_{k=1}^{m} E_0 \exp[tm^{-1}(z_k - T)] \end{aligned} \tag{3}$$

$$\begin{aligned} P_1(f,g,n) &= \Pr[m^{-1} \sum_{k=1}^{m} z_k \le T \mid H_1] \\ &\le E_1 \exp[-t(m^{-1} \sum_{k=1}^{m} z_k - T)] = \\ &= \prod_{k=1}^{m} E_1 \exp[-tm^{-1}(z_k - T)] \end{aligned} \tag{4}$$

for $t \ge 0$, E_j = expectation under H_j. Note that the bound (3) is less that 1 for some $t \ge 0$, if and only if:

$$m^{-1} \sum_{k=1}^{m} E_0 z_k < T \tag{5}$$

Similarly, (4) is less than 1 for some $t \ge 0$, if and only if:

$$m^{-1} \sum_{k=1}^{m} E_1 z_k > T \tag{6}$$

Let, now:

$$z_k = n^{-1} \log [g_{1k}(x_k^n)/g_{0k}(x_k^n)] \tag{7}$$

be the mismatched log-likelihood function for the kth sensor, and $s = t \cdot n^{-1} \ge 0$. Straightforward calculation provides us with the following expressions:

$$E_0 \exp[tm^{-1}(z_k - T)] = \int f_{0k}(x_k^n)[g_{1k}(x_k^n)/g_{0k}(x_k^n)]^{s/m}$$

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$$dx_k^n \cdot \exp(-snT/m). \tag{8}$$

$$E_1 \exp[-tm^{-1}(z_k - T)] = \int f_{1k}(x_k^n)[g_{1k}(x_k^n)/g_{0k}(x_k^n)]^{-s/m}$$

$$dx_k^n \cdot \exp(snT/m) \tag{9}$$

$$E_0\, z_k = n^{-1} \int f_{0k}\,(x_k^n) \log [g_{1k}(x_k^n)/g_{0k}(x_k^n)] dx_k^n \tag{10}$$

$$E_1\, z_k = n^{-1} \int f_{1k}\,(x_k^n) \log [g_{1k}(x_k^n)/g_{0k}(x_k^n)] dx_k^n \tag{11}$$

Define the functionals:

$$G_k\,[f, g_{1k}, g_{0k}, n, s] \stackrel{\Delta}{=}$$

$$n^{-1} \log \int f(x_k^n)[g_{1k}(x_k^n)/g_{0k}(x_k^n)]^{s/m}\, dx_k^n \tag{12}$$

Taking logarithms of (3), (4) and using (8), (9), (12), we find the bounds:

$$n^{-1} \log P_0(f,g,n) \le \sum_{k=1}^{m} G_k[f_{0k}, g_{1k}, g_{0k}, n, s] - sT. \tag{13}$$

$$n^{-1} \log P_1(f,g,n) \le \sum_{k=1}^{m} G_k[f_{1k}, g_{1k}, g_{0k}, n, -s] + sT. \tag{14}$$

We observe that if G_k^1 denotes the derivative of G_k with respect to s, then, from (10), (12), (7):

$$G_k^1[f_{0k}, g_{1k}, g_{0k}, n, 0] = m^{-1}\, E_0\, z_k. \tag{15}$$

$$G_k^1[f_{1k}, g_{1k}, g_{0k}, n, 0] = m^{-1}\, E_1\, z_k. \tag{16}$$

Hence, if (5) is satisfied, the slope of the upper bound (13) at s=0 is negative, and the bound is zero at s=0. Similarly, if (6) is satisfied, the slope of the upper bound (14) at s=0 is negative, and the bound is zero at s=0. Also, both bounds (13) and (14) are convex with respect to s.

The tightened Chernoff bounds (13), (14) are:

$$n^{-1} \log P_0(f,g,n) \le \inf_s \sum_{k=1}^{m} G_k[f_{0k}, g_{1k}, g_{0k}, n, s] - sT. \tag{17}$$

$$n^{-1} \log P_1(f,g,n) \le \inf_s \sum_{k=1}^{m} G_k[f_{1k}, g_{1k}, g_{0k}, n, -s] + sT \tag{18}$$

Suppose, now, that for the class of statistical models we consider, the limits:

$$\lim_{n\to\infty} G_k[f_{jk}, g_{1k}, g_{0k}, n, s] \stackrel{\Delta}{=} G_k[f_{jk}, g_{1k}, g_{0k}, \infty, s] \tag{19}$$

exist, for k=1,...,m, j=0,1

Suppose, also that:

$$\lim_{n\to\infty} n^{-1} \sum_{k=1}^{m} \int f_{0k}\,(x_k^n) \log [g_{1k}(x_k^n)/g_{0k}(x_k^n)] dx_k^n < T \tag{20}$$

$$\lim_{n\to\infty} n^{-1} \sum_{k=1}^{m} \int f_{1k}(x_k^n) \log [g_{1k}(x_k^n)/g_{0k}(x_k^n)] dx_k^n > T \tag{21}$$

Then, utilizing the results of Large Deviation Theory, [8], [9], we find:

$$\lim_{n\to\infty} n^{-1} \log P_0(f,g,n) =$$

$$\inf_s \sum_{k=1}^{m} G_k[f_{0k}, g_{1k}, g_{0k}, \infty, s] - sT. \tag{22}$$

$$\lim_{n\to\infty} n^{-1} \log P_1(f,g,n) =$$

$$\inf_s \sum_{k=1}^{m} G_k[f_{1k}, g_{1k}, g_{0k}, \infty, -s] + sT \tag{23}$$

It is interesting to observe that the bounds (17), (18) become the exact asymptotic error probabilities as $n\to\infty$. This is the essence of Large Deviation Theory [8], [9].

We will now concentrate on a major class of statistical models for which the limits (19), (20), (21) exist. This is the class of Gaussian, stationary random processes with zero means and different spectra. Consider three multivariate Gaussian probability density functions $f(x^n)$, $g(x^n)$, $g_0(x^n)$ with zero means and covariance matrices F, C_1, C_0. We calculate the integral of eq (12):

$$G[f, g_1, g_0, n, \theta] = n^{-1} \log \int f(x^n)[g_1(x^n)/g_0(x^n)]^{\theta} dx^n =$$

$$= -\frac{1}{2}[n^{-1} \log |F_1^{-1} + \theta C_1 - \theta C_0^{-1}| - n^{-1} \log |F_1^{-1}| -$$

$$- \theta \cdot n^{-1} \log |C_1^{-1}| + \theta \cdot n^{-1} \log |C_0^{-1}|\Big] \tag{24}$$

Taking the limit as $n\to\infty$, and using the results of [10], [11], we find:

$$-2G[f, g_1, g_0, \infty, \theta] = (2\pi)^{-1} \int_{-\pi}^{\pi} \{\log[f^{-1}(\lambda) + \theta c_1^{-1}(\lambda) - \theta c_0^{-1}(\lambda)] -$$

$$- \log f^{-1}(\lambda) - \theta \log c_1^{-1}(\lambda) + \theta \log c_0^{-1}(\lambda)\} d\lambda \tag{25}$$

where, $f(\lambda), c_1(\lambda), c_0(\lambda)$ are the three spectra corresponding to f, g_1, g_0, and it is assumed that they are strictly positive for all $\lambda \varepsilon [-\pi, \pi]$. Using (25) for k=1,...,m, and assuming that for the kth sensor the true spectra are $\{f_{0k}(\lambda), f_{1k}(\lambda)\}$ under H_0, H_1, and the assumed spectra are $\{g_{0k}(\lambda), g_{1k}(\lambda)\}$ under H_0, H_1, we can evaluate the rates in (22), (23).

The necessary and sufficient conditions in order for $P_0(f,g,n)$ and $P_1(f,g,n)$ to converge exponentially to zero, are that the derivatives of (22), (23) at s=0 are negative, which are equivalent to (20), (21). The derivative of G with respect to θ at θ=0, is:

$$-2G'[f, g_1, g_0, \infty, \theta=0] = (2\pi)^{-1} \int_{-\pi}^{\pi} \{f(\lambda)[c_1^{-1}(\lambda) - c_0^{-1}(\lambda)] -$$

$$- \log c_0(\lambda) c_1^{-1}(\lambda)\} d\lambda. \tag{26}$$

The condition of negative slope at s=0 of (22) is:

$$(2\pi)^{-1} \int_{-\pi}^{\pi} \sum_{k=1}^{m} \{f_{0k}(\lambda)[g_{1k}^{-1}(\lambda) - g_{0k}^{-1}(\lambda)] -$$

$$\log g_{0k}(\lambda) g_{1k}^{-1}(\lambda)\} d\lambda + 2T > 0 \tag{27}$$

The condition of negative slope at s=0 of (23) is:

$$(2\pi)^{-1}\int_{-\pi}^{\pi}\sum_{k=1}^{m}\{f_{1k}(\lambda)[g_{1k}^{-1}(\lambda)-g_{0k}^{-1}(\lambda)]-$$

$$\log g_{0k}(\lambda)g_{1k}^{-1}(\lambda)\}d\lambda-2T>0, \qquad (28)$$

Conditions (27), (28) guarantee exponential convergence of P_0, P_1 to zero. correspondingly. It can be shown that if (27) is not satisfied, then P_0 converges to 1 as $n \to \infty$. Similarly, if (28) is not satisfied, P_1 converges to 1. The proof of the latter fact is not given here, but it will appear in the expanded version of the paper.

Let us define the spectral distance measure:

$$I(f,g)=(2\pi)^{-1}\int_{-\pi}^{\pi}\{f(\lambda)g^{-1}(\lambda)-1-$$

$$\log f(\lambda)\, g^{-1}(\lambda)\}d\lambda \qquad (29)$$

Because of the identity $x-1 \geq \log x$, it is seen that $I(f,g) \geq 0$ with equality if and only if $f(\lambda)=g(\lambda)$ for almost all $\lambda \varepsilon [-\pi,\pi]$. After some algebraic manipulations, condition (27) is expressed as:

$$\sum_{k=1}^{m} I(f_{0k},g_{1k})-I(f_{0k},g_{0k})+2T>0 \qquad (30)$$

Similarly, condition (28) is expressed as:

$$\sum_{k=1}^{m} I(f_{1k},g_{1k})-(f_{1k},g_{0k})-2T>0 \qquad (31)$$

Combining (30) and (31), we see that the necessary and sufficient condition for exponential convergence of the error rate to zero, is the satisfaction of the double inequality:

$$2^{-1}\sum_{k=1}^{m} I(f_{1k},g_{1k})-I(f_{1k},g_{0k})>T>$$

$$>2^{-1}\sum_{k=1}^{m} I(f_{0k},g_{0k})-I(f_{0k},g_{1k}). \qquad (32)$$

As long as the leftmost side of (32) is larger than the rightmost side, we can always pick a threshold T between those two numbers to achieve asymptotic convergence of the error rates to zero. For the special case of matched statistics, we have $f_{1k}=g_{1k}$, $f_{0k}=g_{0k}$, and the condition (32) becomes:

$$2^{-1}\sum_{k=1}^{m} I(f_{1k},f_{0k})>T>-2^{-1}\sum_{k=1}^{m} I(f_{0k},f_{1k}) \qquad (33)$$

We note that (33) can always be satisfied for some T, because the leftmost side is nonnegative, and the rightmost side nonpositive. Thus, in the "matched" case, exponential convergence hinges only upon the choice of T.

To evaluate the actual rates of convergence, we need to use expression (25) for k=1,...,m into (22), (23) and minimize with respect to s.

CONCLUSIONS

We have obtained the rates of convergence of error probabilities in multisensor detection for a binary hypothesis, and we determined the conditions of exponential convergence in the presence of mismatch. The conditions are necessary and sufficient.

REFERENCES

[1] Tenney, R. R. and Sandell, N. R. (1981) "Detection with distributed sensors," IEEE Transactions on AES, July 1981, pp. 501-510.

[2] Teneketzis, D. (1982) "The decentralized quickest detection problem," Proceedings of the 21st IEEE Conf. on Decision and Control.

[3] Srinivasan, R (1986), "Distributed Radar Detection Theory," IEE Proceedings, vol 133, ptF, Feb 1986, pp. 55-60.

[4] Chair, Z and Varshney, P. (1987) "Neymann-Pearson Hypothesis Testing in Distributed Networks," 1987 Conf. on Decision and Control.

[5] Reibman, A. R. and Nolte, L. W. (1987) "Optimal Detection and Performance of Distributed Sensor Systems," IEEE Trans. on AES, vol 23, no 1, pp. 24-30.

[6] Viswanathan, R. Thomopoulos, S.C.A., Tumulurı, R., "Spatial Serial Distributed Decision Fusion," 1987 IEEE Conf. on Decision and Control.

[7] Reibman, A. R. and Nolte, L. W., "Design and Performance Comparison of Distributed Detecton Networks," IEEE Trans. on AES, Vol 23, no 6, Nov 1987, pp 789-797.

[8] Chernoff, H. (1952), "A measure of asymptotic efficiency for tests of a hypothesis based on the sum of observations," Ann. of Math. Stat. vol 23, pp. 493-507, 1952.

[9] Ellis, R. S. (1984), "Large Deviations for a General Class of Random Vectors," Annals of Prob., vol 12, pp 1-12, 1984.

[10] Kazakos, D. and Papantoni-Kazakos, P. (1980), "Spectral Distance Measures Between Gaussian Processes," IEEE Trans. on Aut. Control, vol AC-25, no 5, October 1980, pp 950-959.

[11] Kazakos, D. (1982), "Statistical Discrimination Using Inaccurate Models," IEEE Trans. on Infor. Theory, Sept 1982, vol IT-28.

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